Fuzzy topology on fuzzy sets: fuzzy semicontinuity and fuzzy semiseparation axioms

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Abstract

The concept of a fuzzy topology on a fuzzy set was introduced in [Fuzzy Set Syst. 45 (1992) 103]. The aim of this paper is to introduce fuzzy semicontinuity and fuzzy semiseparation axioms in this new situation and validity of some characterization of these concepts have been examined. Also, we define a fuzzy generalized semiopen set and introduce fuzzy separation axioms by using this concept. Fuzzy semiconnected and fuzzy semicompact spaces are introduced and some of their properties are discussed.

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1. Introduction

Chakrabarty and Ahsanullah [4] introduced the concept of a fuzzy topology on a fuzzy set and defined the category FUZZ-TOP where the objects are F-ts \((\lambda, \delta)\) and the morphisms are F-continuous proper functions. There they studied neighbourhood systems, \(q\)-neighbourhood systems and subspaces of such F-ts. They extended the notion of \(q\)-coincidence to fit in this new situation. Chaudhuri and Das [6] studied several fundamental properties of such
fuzzy topologies. The $\delta$-continuity, fuzzy almost regularity (normality) was introduced by Zahran [13]. In this paper, we introduce the concepts of a fuzzy semipoint and fuzzy semicontinuity in this new situation and we examine the validity of the standard results. Also, we introduce fuzzy semiseparation axioms and we give a characterization of a fuzzy semiregular space. The concept of a fuzzy generalized semipoint set is defined and fuzzy separation axioms are studied by using this concept. The concepts of fuzzy semiconnected, and fuzzy semicompact spaces are discussed.

For definitions and results not explained in this paper, we refer to [2,4–9] assuming them to be well known.

2. Preliminaries

In this paper, the symbol $I$ will denote the closed unit interval, let $X$ be a non-empty set. A fuzzy set of $X$ is a function with domain $X$ and values in $I$, that is, an element of $I^X$. A fuzzy point $x_r$ is a fuzzy set [11] whose support is the point $x$ and its value $0 < r \leq 1$, we write $x_r \in \lambda(x) \in \lambda$ if $r \leq \lambda(x)$ ($r < \lambda(x)$). In this paper, we will shorten the word fuzzy as F [9]. The family of all F-points (resp. F-semipoint sets, F-semiclosed sets, F-generalized semipoint sets, F-generalized semiclosed sets) of $\lambda$ will be denoted by $Pt(\lambda)$ (resp. $FSO(\lambda)$, $FSC(\lambda)$, $FGSO(\lambda)$, $FGSC(\lambda)$).

**Definition 2.1** [4]. If $\rho \in \lambda$, the complement of $\rho$ referred to $\lambda$, denoted by $\rho'$, is defined by $\rho'_r(x) = \lambda(x) - \rho(x), \forall x \in X$.

**Remark 2.2.** Let $\lambda \in I^X, \lambda = \{ e \in I^X : e \leq \lambda \}$ and $\mu \in I^Y, \lambda = \{ e \in I^X : e \leq \mu \}, \forall \lambda, \mu \in I^X, \lambda$ is an F-lattice (completely distributive lattice) which has an order-reversing involution $\lambda : \lambda \rightarrow \lambda$.

**Definition 2.3** [4]. $\rho, \sigma \in \lambda$ are said to be quasi-coincident to $\lambda$ [written as $\rho \equiv \sigma[\lambda]$] if there exists $x \in X$ such that $\rho(x) + \sigma(x) > \lambda(x)$. If $\rho$ and $\sigma$ are not quasi-coincident referred to $\lambda$, we denote for this $\rho \equiv \sigma[\lambda]$.

**Definition 2.4** [4]. A F-subset $f$ of $X \times Y$ is said to be a F-proper function from $\lambda(\in I^X)$ to $\mu(\in I^Y)$ if

(i) $f(x, y) \leq \lambda(x) \land \mu(y), \forall (x, y) \in X \times Y$.

(ii) $\forall x \in X, \exists y \in Y$ such that $f(x, y) = \lambda(x)$ and $f(x, y) = 0$ if $y \neq y_0$.

**Definition 2.5.** Let $f : \lambda \rightarrow \mu$ be a F-proper function from $\lambda$ to $\mu$. Based on $f : \lambda \rightarrow \mu$ (and adopting Rodabaugh’s symbols [12]), define the correspondent
F-proper function \( f^\leftarrow : \mathcal{A}_\lambda \rightarrow \mathcal{B}_\mu \) and its reverse F-proper function \( f^\rightarrow : \mathcal{B}_\mu \rightarrow \mathcal{A}_\lambda \) by

(i) \( f^\leftarrow : \mathcal{A}_\lambda \rightarrow \mathcal{B}_\mu, f^\leftarrow (\rho)(y) = \vee\{f(x,y) \wedge \rho(x) : x \in X\}, \forall \rho \in \mathcal{A}_\lambda, \forall y \in Y. \)

(ii) \( f^\rightarrow : \mathcal{B}_\mu \rightarrow \mathcal{A}_\lambda, f^\rightarrow (\sigma)(x) = \vee\{f(x,y) \wedge \sigma(y) : y \in Y\}, \forall \sigma \in \mathcal{B}_\mu, \forall x \in X. \)

**Definition 2.6** [6]. If \( \sigma \in \mathcal{A}_\lambda \) is said to be maximal if \( \forall x \in X, \sigma(x) \neq 0, \) then \( \sigma(x) = \lambda(x) \).

**Proposition 2.7** [6]. If \( \sigma \in \mathcal{B}_\mu \) is maximal, then \( f^\leftarrow (\sigma'_\mu) = (f^\rightarrow (\sigma))'_\lambda \).

**Proposition 2.8** [6]. For a F-proper function \( f : \lambda \rightarrow \mu \), Then

(i) \( f^\leftarrow (f^\rightarrow (\sigma)) \leq \sigma, \forall \sigma \in \mathcal{B}_\mu. \)

(ii) \( f^\leftarrow (f^\rightarrow (\rho)) \geq \rho, \forall \rho \in \mathcal{A}_\lambda. \)

(iii) \( f^\leftarrow (\zeta \vee \sigma) = f^\leftarrow (\zeta) \vee f^\leftarrow (\sigma) \) and in general \( f^\leftarrow (\bigvee_{j \in J} \sigma_j) = \bigvee_{j \in J} f^\leftarrow (\sigma_j), \forall \zeta, \sigma_j \in \mathcal{B}_\mu. \)

(iv) \( f^\leftarrow (\zeta \wedge \sigma) = f^\leftarrow (\zeta) \wedge f^\leftarrow (\sigma), \forall \zeta, \sigma \in \mathcal{B}_\mu. \)

**Definition 2.9** [4]. A collection \( \delta \) of F-subsets of \( \lambda \) i.e \( \delta \subset \mathcal{A}_\lambda \) is said to be a fuzzy topology on \( \lambda \) if

(i) \( 0, \lambda \in \delta. \)

(ii) \( \rho_j \in \delta \Rightarrow \bigvee_j \rho_j \in \delta. \)

(iii) \( \rho, \sigma \in \delta \Rightarrow \rho \wedge \sigma \in \delta. \)

\((\lambda, \delta)\) is said to be a fuzzy topological space (briefly, F-ts). The members of \( \delta \) are said to be F-open sets of \( \lambda \).

We denote \( \delta' \) the family of F-closed sets of \( \lambda \), that is, \( \rho \in \delta' \) iff \( \lambda - \rho \in \delta \).

**Remark 2.10.** A fuzzy topology on a fuzzy set can not be extended to the \( L \)–fuzzy setting. For this see Remark 3.1.7 in [9].

**Definition 2.11.** Let \((\lambda, \delta)\) be a F-ts and \( \rho \in \mathcal{A}_\lambda \). Then the interior (closure) of \( \rho \) is defined by

(i) \( \text{Int}(\rho) = \bigvee\{\zeta : \zeta \in \delta, \zeta \leq \rho\} \) [13].

(ii) \( \text{Cl}(\rho) = \bigwedge\{\eta : \eta \in \delta', \rho \leq \eta\} \) [4].

**Proposition 2.12** [13]. Let \( \rho \) and \( \sigma \) be F-subsets of a F-ts \((\lambda, \delta)\). Then

\( \text{Int}(\rho'_\lambda) = (\text{Cl}(\rho))'_\lambda \) and \( \text{Cl}(\sigma'_\lambda) = (\text{Int}(\sigma))'_\lambda. \)
Proposition 2.13. In a F-ts $(X, \delta)$, where $X$ is an ordinary set, for a function $f : (X, \delta) \to (Y, \delta^*)$, if $f$ is F-semicontinuous, then the following are true

(i) $f^-\sigma$ is F-semiclosed, $\forall \sigma \in \delta^*$ [2].

(ii) $f^-\text{Scl}(\rho) \subseteq \text{Cl}(f^-\rho)$, $\forall$ F-subset $\rho$ of $X$ [3].

Theorem 2.14 [10]. In a F-ts $(X, \delta)$, where $X$ is an ordinary set for a function $f : (X, \delta) \to (Y, \delta^*)$, then $f$ is F-irresolute if $f^-\sigma \in \text{FSC}(X)$, $\forall \sigma \in \text{FSC}(Y)$.

Proposition 2.15 [8]. Let $f_1 : \lambda_1 \to \mu_1$, $f_2 : \lambda_2 \to \mu_2$ be F-proper functions, $f_1 \times f_2 : \lambda_1 \times \lambda_2 \to \mu_1 \times \mu_2$ be the product F-proper function. Then

(i) If $\rho \leq \lambda_1, \sigma \leq \lambda_2$, then $(f_1 \times f_2)^-\rho \times \sigma = f_1^-\rho \times f_2^-\sigma$.

(ii) If $\zeta \leq \mu_1, \eta \leq \mu_2$, then $(f_1 \times f_2)^-\zeta \times \eta = f_1^-\zeta \times f_2^-\eta$.

Definition 2.16 [1]. A F-subset $\rho$ of a F-ts $(X, \delta)$ is said to be

(i) Fg-open if $\forall \eta \in \delta', \eta \leq \rho$, then $\eta \leq \text{Int}(\rho)$.

(ii) Fg-closed if $\forall \zeta \in \delta, \rho \leq \zeta$, then $\text{Cl}(\rho) \leq \zeta$.

Theorem 2.17 [3]. A F-ts $(X, \delta)$ is FS-T1 iff each F-point is F-semiclosed.

Definition 2.18 [6]. A F-ts $(\lambda, \delta)$ is said to be F-connected if it has no proper non-zero maximal F-clopen set.

3. Fuzzy semicontinuity

Definition 3.1. Let $\rho$ be a F-subset of a F-ts $(\lambda, \delta)$. Then $\rho$ is said to be

(i) F-semiopen if $\rho \leq \text{Cl}(\text{Int}(\rho))$.

(ii) F-semiclosed if $\rho \geq \text{Int}($Cl$(\rho))$.

Proposition 3.2. The complement of a F-semiopen set is a F-semiclosed set.

Remark 3.3. Each F-open (resp. F-closed) set is a F-semiopen (resp. F-semiclosed), but the converse need not be true in general as shown by the following example.

Example 3.4. Let $X = \{x, y\}, \lambda = x_{0.7} \vee y_{0.6}, \delta = \{0, x_{0.4} \vee y_{0.3}, x_{0.6} \vee y_{0.4}, \lambda\}$. It is clear that, $x_{0.5} \vee y_{0.4} \in \text{FSO}(\lambda)$, but $x_{0.5} \vee y_{0.4} \notin \delta$. Also, $0.2 \in \text{FSC}(\lambda)$, but $0.2 \notin \delta'$.
Proposition 3.5
(i) Any union of $F$-semiopen sets is a $F$-semiopen set.
(ii) Any intersection of $F$-semiclosed sets is a $F$-semiclosed set.

Remark 3.6
(i) The intersection of $F$-semiopen sets is not necessary $F$-semiopen set.
(ii) The union of $F$-semiclosed sets is not necessary $F$-semiclosed set.

Example 3.7. Let $X = \{x, y\}$, $\lambda = x_{0.6} \cup y_{0.59}$, $\delta = \{0, 0.4, x_{0.51} \cup y_{0.1}, x_{0.1} \cup y_{0.51}, 0.1, x_{0.1} \cup y_{0.4}, x_{0.4} \cup y_{0.1}, 0.51, x_{0.4} \cup y_{0.51}, x_{0.51} \cup y_{0.4}, \lambda\}$.

It is clear that, $x_{0.3} \cup y_{0.51}, x_{0.51} \cup y_{0.3} \in \text{FSO}(\lambda)$, but $0.3 \notin \text{FSO}(\lambda)$. Also, $x_{0.3} \cup y_{0.08}, x_{0.08} \cup y_{0.3} \in \text{FSC}(\lambda)$, but $0.3 \notin \text{FSC}(\lambda)$.

Definition 3.8. Let $(\lambda, \delta)$ be a $F$-ts. Then $F$-semiclosure $(\text{Scl})$ and $F$-semiinterior $(\text{Sint})$ of a $F$-subset $\rho$ of $\mathcal{A}_\lambda$ are defined as follows:

(i) $\text{Scl}(\rho) = \bigwedge\{\xi \in \mathcal{A}_\lambda : \xi \in \text{FSC}(\lambda), \rho \subseteq \xi\}$.
(ii) $\text{Sint}(\rho) = \bigvee\{\eta \in \mathcal{A}_\lambda : \eta \in \text{FSO}(\lambda), \eta \subseteq \rho\}$.

Proposition 3.9. Let $(\lambda, \delta)$ be a $F$-ts. Then.

(i) $\text{Scl}(\emptyset) = 0$.
(ii) $\forall \rho \in \mathcal{A}_\lambda$, $\text{Scl}(\rho)$ is $F$-semiclosed.
(iii) $\forall \rho \in \mathcal{A}_\lambda$, $\rho \subseteq \text{Scl}(\rho)$.
(iv) $\forall \rho, \sigma \in \mathcal{A}_\lambda$, $\rho \subseteq \sigma \Rightarrow \text{Scl}(\rho) \subseteq \text{Scl}(\sigma)$.
(v) $\forall \rho \in \mathcal{A}_\lambda$, $\text{Scl}(\text{Scl}(\rho)) = \text{Scl}(\rho)$.

Proposition 3.10. In a $F$-ts $(\lambda, \delta)$, $\rho \in \mathcal{A}_\lambda$ is a $F$-semiclosed (resp. $F$-semiopen) set iff $\rho = \text{Scl}(\rho)$ (resp. $\rho = \text{Sint}(\rho)$).

Proposition 3.11. Let $(\lambda, \delta)$ be a $F$-ts. Then

(i) $\forall \rho \in \mathcal{A}_\lambda$, $\text{Scl}(\rho') = (\text{Scl}(\rho'))'$.
(ii) $\forall \rho \in \mathcal{A}_\lambda$, $\text{Sint}(\rho') = (\text{Scl}(\rho'))'$.

Proposition 3.12. Let $(\lambda, \delta)$ be a $F$-ts. Then

(i) $\forall \zeta, \eta \in \mathcal{A}_\lambda$, $\text{Scl}(\zeta \cup \eta) \supseteq \text{Scl}(\zeta) \cup \text{Scl}(\eta)$.
(ii) $\forall \zeta, \eta \in \mathcal{A}_\lambda$, $\text{Scl}(\zeta \wedge \eta) \subseteq \text{Scl}(\zeta) \wedge \text{Scl}(\eta)$.
(iii) If $\eta \in \text{FSO}(\lambda)$, then $\eta \nq \zeta \Leftrightarrow \zeta \leq \eta'$.

Proof. (i) and (ii) are clear, so we need only prove (iii) Let $\eta \in \text{FSO}(\lambda)$ and $\zeta \in \mathcal{A}_\lambda$ such that $\eta \nq \zeta \Leftrightarrow \zeta \leq \eta' \Leftrightarrow \text{Scl}(\zeta) \leq \eta' \Leftrightarrow \eta \nq \text{Scl}(\zeta) \Leftrightarrow \Box$.
**Proposition 3.13.** Let \((\lambda, \delta)\) be a F-ts. Then:

(i) \(\text{Sint}(0) = 0\).
(ii) \(\forall \rho \in \mathcal{A}_\lambda, \text{Sint}(\rho) \) is F-semiopen.
(iii) \(\forall \rho \in \mathcal{A}_\lambda, \text{Sint}(\rho) \leq \rho\).
(iv) \(\forall \rho, \sigma \in \mathcal{A}_\lambda, \rho \leq \sigma \Rightarrow \text{Sint}(\rho) \leq \text{Sint}(\sigma)\).
(v) \(\forall \rho \in \mathcal{A}_\lambda, \text{Sint}(\text{Sint}(\rho)) = \text{Sint}(\rho)\).

**Proposition 3.14.** Let \((\lambda, \delta)\) be a F-ts. Then

(i) \(\forall \zeta, \eta \in \mathcal{A}_\lambda, \text{Sint}(\zeta \lor \eta) \geq \text{Sint}(\zeta) \lor \text{Sint}(\eta)\).
(ii) \(\forall \zeta, \eta \in \mathcal{A}_\lambda, \text{Sint}(\zeta \land \eta) \leq \text{Sint}(\zeta) \land \text{Sint}(\eta)\).

**Definition 3.15.** A F-proper function \(f : (\lambda, \delta) \rightarrow (\mu, \delta')\) is said to be

(i) F-semicontinuous if \(f^{-1}(\sigma) \in \text{FSO}(\lambda), \forall \sigma \in \delta'\).
(ii) F-irresolute if \(f^{-1}(\sigma) \in \text{FSO}(\lambda), \forall \sigma \in \text{FSO}(\mu)\).
(iii) F-semiopen if \(f^{-1}(\rho) \in \text{FSO}(\mu), \forall \rho \in \delta\).
(iv) F-semiclosed \(f^{-1}(\rho) \in \text{FSC}(\mu), \forall \rho \in \delta'\).

**Remark 3.16.** From the above definitions one may notice that: F-continuous (F-irresolute) \(\Rightarrow\) F-semicontinuous.

**Proposition 3.17**

(i) \(f^{-1}(\sigma) \in \text{FSC}(\lambda), \forall \sigma \in \delta''\).
(ii) \(f^{-1}(\text{Scl}(\rho)) \leq \text{Cl}(f^{-1}(\rho)), \forall \rho \in \mathcal{A}_\lambda\).

*The conditions (i) and (ii) are neither necessary nor sufficient for \(f\) to be F-semicontinuous; i.e. Proposition 2.13 is not true in a F-ts(\(\lambda, \delta\)).*

**Proposition 3.18.** If \(\sigma\) is a maximal F-closed (resp. F-semiclosed) subset of \(\lambda\), then \(f\) is F-semicontinuous (resp. F-irresolute) iff \(f^{-1}(\sigma) \in \text{FSC}(\lambda), \forall \sigma \in \delta''\) (resp. \(f^{-1}(\sigma) \in \text{FSC}(\lambda), \forall \lambda \in \text{FSC}(\mu)\)).

**Proposition 3.19.** Let \((\lambda, \delta_1), (\mu, \delta_2)\) and \((v, \delta_3)\) are F-ts’s. If \(f : (\lambda, \delta_1) \rightarrow (\mu, \delta_2)\) and \(g : (\mu, \delta_2) \rightarrow (v, \delta_3)\) are F-proper functions, then

(i) If \(g\) is F-continuous and \(f\) is F-semicontinuous, then \(gf\) is F-semicontinuous.
(ii) If \(g\) is F-semicontinuous and \(f\) is F-irresolute, then \(gf\) is F-semicontinuous.
(iii) If \(g\) is F-semiopen and \(f\) is F-open, then \(gf\) is F-semiopen.

**Theorem 3.20.** Let \(f : (\lambda, \delta) \rightarrow (\mu, \delta')\) be a F-proper function from \(\lambda\) to \(\mu\). Then the following are equivalent:
(i) \( f \) is F-semicontinuous.

(ii) \( \forall x, y \in \mathcal{P}(\lambda), \forall \varepsilon > 0 \) such that \( f(x, y) \neq 0 \) and \( \forall \sigma \in \delta^* \) with \( y \in \sigma \), let \( \exists \rho \in \text{FSO}(\lambda) \) such that \( x \in \rho \) and \( f^-(\rho) \leq \varepsilon \).

(iii) \( \forall x, y \in \mathcal{P}(\lambda), \forall \varepsilon > 0 \) such that \( f(x, y) \neq 0 \) and \( \forall \sigma \in \delta^* \) with \( y \notin \sigma \), let \( \exists \rho \in \text{FSO}(\lambda) \) such that \( x \notin \rho \) and \( f^-(\rho) \leq \varepsilon \).

(iv) \( \forall x, y \in \mathcal{P}(\lambda), \forall \varepsilon > 0 \) such that \( f(x, y) \neq 0 \) and \( \forall q \text{-nbd} \sigma \) of \( y \), let \( \exists \rho \in \text{FSO}(\lambda) \) such that \( x \notin \rho \) and \( f^-(\rho) \leq \varepsilon \).

(v) \( \forall x, y \in \mathcal{P}(\lambda), \forall \varepsilon > 0 \) such that \( f(x, y) \neq 0 \) and \( \forall q \text{-nbd} \sigma \) of \( y \), let \( \exists \rho \in \text{FSO}(\lambda) \) such that \( x \notin \rho \) and \( f^-(\rho) \leq \varepsilon \).

Remark 3.21. If \( f_1 : \lambda_1 \rightarrow \mu_1 \) and \( f_2 : \lambda_2 \rightarrow \mu_2 \) are F-semicontinuous functions, then \( f_1 \times f_2 : \lambda_1 \times \lambda_2 \rightarrow \mu_1 \times \mu_2 \) may not be F-semicontinuous.

Remark 3.22. The following example shows that Theorem 2.14 is not true in fuzzy topology on fuzzy set.

Example 3.23. Let \( X = \{x, y\}, Y = \{a, b\}, \lambda = 0.7 \) and \( \mu = 0.8 \). Consider the fuzzy topologies \( \delta = \{0, 0.1, 0.2, 0.3\} \) on \( \lambda \) and \( \delta^* = \{0, 0.2, 0.4\} \) on \( \mu \).

Let \( f : (\lambda, \delta) \rightarrow (\mu, \delta^*) \) defined by \( f(x, a) = 0.7, f(y, b) = 0.7, f(x, b) = f(y, a) = 0 \). It is clear that, \( f^-(\sigma) \) is not F-irresolute.

4. Fuzzy semi-separation axioms

Definition 4.1. A F-ts \((\lambda, \delta)\) is said to be

(i) FS-T_0 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \), there exists \( \zeta \in \text{FSO}(\lambda) \) such that \( x \in \zeta, \zeta^\cap \hat{q} y [\lambda] \) or \( y \in \zeta, \zeta^\cap \hat{q} x [\lambda] \).

(ii) FS-T_1 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \), there exist \( \zeta, \eta \in \text{FSO}(\lambda) \) such that \( x \in \zeta, \zeta^\cap \hat{q} y [\lambda] \) and \( y \in \eta, \eta^\cap \hat{q} x [\lambda] \).

(iii) FS-T_2 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \), there exist \( \zeta, \eta \in \text{FSO}(\lambda) \) such that \( x \in \zeta, \eta^\cap \hat{q} x [\lambda] \) and \( y \in \eta, \eta^\cap \hat{q} y [\lambda] \).

(iv) FS-T_3 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \), there exist \( \zeta, \eta \in \text{Scl}(\zeta) \) such that \( x \in \zeta, \zeta^\cap \hat{q} y [\lambda] \) and \( y \in \eta, \eta^\cap \hat{q} x [\lambda] \).

(v) FS-R_0 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \), and each \( \zeta \in \text{FSO}(\lambda) \) such that \( x \in \zeta, \zeta^\cap \hat{q} y [\lambda] \), there exists \( \eta \in \text{FSO}(\lambda) \) such that \( y \in \eta, \eta^\cap \hat{q} x [\lambda] \).

(vi) FS-R_1 if for each \( x, y \in \mathcal{P}(\lambda) \) with distinguished support points \( x \neq y \) and each \( \zeta \in \text{FSO}(\lambda) \) such that \( x \in \zeta, \zeta^\cap \hat{q} y [\lambda] \), there exist \( \rho, \sigma \in \text{FSO}(\lambda) \) such that \( x \in \rho, y \in \sigma \) and \( \rho^\cap \hat{q} \sigma [\lambda] \).

Remark 4.2. From the above definition one may notice that:

(i) FS-T_2 \( \Rightarrow \) FS - T_2 \( \Rightarrow \) FS - T_1 \( \Rightarrow \) FS - T_0.

(ii) FS-R_1 \( \Rightarrow \) FS - R_0.
Example 4.3. Let \( X = \{x, y\}, \lambda = x_{0.6} \lor y_{0.5}, \delta = \{0, x_{0.6} \lor y_{0.4}, y_{0.4}, \lambda\} \). It is clear that, the F-ts \((\lambda, \delta)\) is FS-T_0 but not FS-T_1.

Remark 4.4. Theorem 2.17 is not true in a F-ts \((\lambda, \delta)\) as shown by the following example.

Example 4.5. Let \( X = \{x, y\}, \lambda = x_{0.6} \lor y_{0.5}, \delta = \{0, x_{0.6}, y_{0.5}, \lambda\} \). It is clear that, the F-ts \((\lambda, \delta)\) is FS-T_1, but the F-point \( y_{0.3} \) is not F-semicolonized.

Theorem 4.6. A F-ts \((\lambda, \delta)\) is FS-T_2 iff \(\forall x \in X\) and \(\lambda(x) > r \land \{\text{Scl}(\rho) : \rho \in \mathcal{A}_x, \lambda \in \text{FSO}(\lambda), x \in \rho\}\) does not contain any F-point \( y \) such that \( x \neq y, s < \lambda(y) \).

Proof. Let \((\lambda, \delta)\) be a FS-T_2 space, \(\lambda(x) > r, x \neq y\) and \(s < \lambda(y)\). Put \( \lambda(y) - s = t \). Now \( x_r, y_s \in Pr(\lambda), \exists \zeta, \eta \in \text{FSO}(\lambda) \) such that \( x_r, y_s \in \eta \) and \( \zeta \in \text{Scl}(\rho) \). Then \( \zeta \leq \eta \Rightarrow \text{Scl}(\zeta) \leq \text{Scl}(\eta) = \eta' \). Since \( y_s \in \eta \), then \( t < \eta(y) \), \( \lambda(y) - t > \eta'(y) \). So, \( y_s \notin \text{Scl}(\zeta) \) and \( y_s \notin \Lambda \{\text{Scl}(\zeta) : \zeta \in \mathcal{A}_x, \lambda \in \text{FSO}(\lambda), x \in \eta \} \).

Conversely, let the given condition hold, \( x_r, y_s \in Pr(\lambda) \). Put \( \lambda(y) - s = t \). By the given condition, \( \exists \rho \in \text{FSO}(\lambda) \) such that \( x_r, y_s \in \rho, y_s \notin \text{Scl}(\rho) \). Put \( \text{Scl}(\rho) = \sigma \), then \( \sigma \in \text{FSO}(\lambda) \). Now we have \( x_r, y_s \in \rho, y_s \in \sigma \) and \( \rho \in \text{FSO}(\lambda) \). So, \((\lambda, \delta)\) is FS-T_2.

Theorem 4.7. If a F-ts \((\lambda, \delta)\) is FS-T_0 and FS-R_1, then \((\lambda, \delta)\) is a FS-T_2 space.

Remark 4.8. If \((\lambda \times \mu, \delta \times \delta')\) is FS-T_2, then \((\lambda, \delta)\) may not be FS-T_2 as shown by the following example.

Example 4.9. Let \( X = \{x, y\}, Y = \{a, b\}, \lambda = x_{0.4} \lor y_{0.5}, \mu = a_{0.3} \lor b_{0.2}, \delta = \{0, x_{0.3}, x_{0.2}, y_{0.1}, y_{0.2}, x_{0.2} \lor y_{0.1}, y_{0.4} \lor y_{0.1}, x_{0.3} \lor y_{0.1}, y_{0.4}, 0.4, x_{0.2} \lor y_{0.4}, \lambda\} \), \( \delta' = \{0, a_{0.3}, b_{0.2}, \mu\} \). It is clear that, the space \((\lambda \times \mu, \delta \times \delta')\) is FS-T_2, but \((\lambda, \delta)\) is not FS-T_2.

Definition 4.10. A F-ts \((\lambda, \delta)\) is said to be F-semiregular if for each \( x_r \in Pr(\lambda), \sigma \in \delta' \) such that \( \sigma \in \text{Scl}(\lambda) \), there exist \( \zeta, \eta \in \text{FSO}(\lambda) \) such that \( x_r \in \zeta, \sigma \leq \eta \) and \( \zeta \in \text{Scl}(\eta) \).

Theorem 4.11. For a F-ts \((\lambda, \delta)\) the following statements are equivalent:

(i) \((\lambda, \delta)\) is F-semiregular;
(ii) For each \( x_r \in Pr(\lambda) \) and each \( \sigma \in \delta' \) such that \( \sigma \in \text{Scl}(\lambda) \), there exist \( \zeta, \eta \in \text{FSO}(\lambda) \) such that \( x_r \in \zeta, \sigma \leq \eta \) and \( \zeta \in \text{Scl}(\eta) \).
(iii) For each \( x_r \in \Pr(\lambda) \) and each \( \rho \in \delta \) such that \( x_r \in \rho \), there exists \( \zeta \in \FSO(\lambda) \) such that \( x_r \in \zeta \leq \Scl(\zeta) \leq \rho \).
(iv) For each \( x_r \in \Pr(\lambda) \) and each \( \sigma \in \delta' \) such that \( \sigma \cap x_r[\lambda] \), there exist \( \zeta, \eta \in \FSO(\lambda) \) such that \( x_r \in \zeta, \sigma \leq \eta \) and \( \Scl(\zeta) \cap \Scl(\eta)[\lambda] \).
(v) For each \( \xi \in \mathcal{A}_\lambda \) and \( \sigma \in \delta' \) such that \( \xi \cap \sigma[\lambda] \), there exist \( \zeta, \eta \in \FSO(\lambda) \) such that \( \xi \subseteq \zeta, \sigma \leq \eta \) and \( \xi \cap \eta \).
(vi) For each \( \xi \in \mathcal{A}_\lambda \) and \( \rho \in \delta \) such that \( \xi \leq \rho \), there exists \( \zeta \in \FSO(\lambda) \) such that \( \xi \leq \zeta \leq \Scl(\zeta) \leq \rho \).

**Proof.** (i) \( \Rightarrow \) (ii), (i) \( \Rightarrow \) (iv), (iv) \( \Rightarrow \) (v), (v) \( \Rightarrow \) (vi) and (vi) \( \Rightarrow \) (i): are clear by Proposition 3.12 (iii).

(ii) \( \Rightarrow \) (iii): Let \( \rho \in \delta, x_r \in \Pr(\lambda) \) and \( x_r \in \rho \). Then \( \rho \cap x_r[\lambda] \). By (ii), \( \exists \zeta, \eta \in \FSO(\lambda) \) such that \( x_r \in \zeta, \rho \leq \eta \) and \( \Scl(\eta) \cap \Scl(\eta)[\lambda] \). Then \( \zeta \cap \eta[\lambda] \Rightarrow \zeta \leq \eta \) \( \Rightarrow \) \( \Scl(\zeta) \leq \Scl(\eta) \). So, \( x_r \in \zeta \leq \Scl(\zeta) \leq \rho \).

(iii) \( \Rightarrow \) (i): Let \( \sigma \in \delta', x_r \in \Pr(\lambda) \) and \( \sigma \cap x_r[\lambda] \). Then \( x_r \in \sigma, \sigma \in \delta \). By (iii), \( \exists \zeta, \eta \in \FSO(\lambda) \) such that \( x_r \in \zeta \subseteq \Scl(\zeta) \subseteq \sigma \). Put \( (\Scl(\zeta))_\lambda = \eta \), then \( \eta \in \FSO(\lambda) \), \( x_r \in \zeta, \sigma \leq \eta \) and \( \zeta \cap \eta[\lambda] \). So, \( (\lambda, \delta) \) is F-semiregular. \( \square \)

**Definition 4.12.** A F-ts \( (\lambda, \delta) \) is said to be a F-semi-normal space if for each \( \sigma_1, \sigma_2 \in \delta' \) such that \( \sigma_1 \cap x_2[\lambda] \), there exist \( \zeta, \eta \in \FSO(\lambda) \) such that \( \sigma_1 \subseteq \zeta, \sigma_2 \subseteq \eta \) and \( \zeta \cap \eta[\lambda] \).

**Theorem 4.13.** A F-ts \( (\lambda, \delta) \) is F-semi-normal iff for each \( \sigma \in \delta', \rho \in \delta \) such that \( \sigma \subseteq \rho \), there exists \( \zeta \in \FSO(\lambda) \) such that \( \sigma \subseteq \zeta \subseteq \Scl(\zeta) \subseteq \rho \).

**Example 4.14.** Let \( X = I, \lambda = 0.9 \) and \( \delta = \{0.0, 0.3, \lambda\} \). It is clear that the F-ts \( (\lambda, \delta) \) is F-semi-normal but not F-semi-normal.

5. Generalized semiseparation axioms

**Definition 5.1.** Let \( (\lambda, \delta) \) be a F-ts, \( \rho \in \mathcal{A}_\lambda \). Then \( \rho \) is said to be

(i) Fg-semiopen if \( \forall \eta \in \FSO(\lambda), \eta \leq \rho \), then \( \eta \leq \Sint(\rho) \).

(ii) Fg-semiclosed if \( \forall \zeta \in \FSO(\lambda), \rho \leq \zeta \), then \( \Scl(\rho) \leq \zeta \).

**Theorem 5.2.** A F-subset is Fg-semiopen iff its complement is Fg-semiclosed.

**Remark 5.3.** One may notice that each F-semiopen (resp. F-semiclosed) set is a Fg-semiopen (resp. Fg-semiclosed) set, but the converse need not be true in general as shown by the following example:
Example 5.4. Let $X = \{x, y\}$, $\lambda = 0.6$ and $\delta = \{0.\overline{2}, 0.4, \lambda\}$. It is clear that, $0.\overline{5} \in \text{FGSO}(\lambda)$, but $0.\overline{5} \notin \text{FSO}(\lambda)$. Also, $0.1 \in \text{FGSC}(\lambda)$, but $0.1 \notin \text{FSC}(\lambda)$.

Theorem 5.5. Let $\rho$ be a Fg-semiopen (resp. Fg-semiclosed) set of a F-ts ($\lambda, \delta$). Then for each F-semiclosed (resp. F-semiopen) set $\zeta$ such that $\zeta \leq \rho$ (resp. $\rho \leq \zeta$), there exists a F-semiopen (resp. F-semiclosed) set $\sigma$ such that $\zeta \leq \sigma \leq \rho$ (resp. $\rho \leq \sigma \leq \zeta$).

Theorem 5.6. Let $\rho$ be a Fg-semiclosed (resp. Fg-semiopen) set of a F-ts ($\lambda, \delta$) and $\rho \leq \sigma \leq \text{Scl}(\rho)$ (resp. $\text{Sint}(\rho) \leq \sigma \leq \rho$). Then $\sigma$ is a Fg-semiclosed (resp. Fg-semiopen) set.

Theorem 5.7. In a F-ts ($\lambda, \delta$), FSO ($\lambda$) = FSC ($\lambda$) iff each F-subset of $\mathcal{A}_\lambda$ is Fg-semiclosed and Fg-semiopen.

Proof. Let $\text{FSO}(\lambda) = \text{FSC}(\lambda)$, $\rho \in \mathcal{A}_\lambda$ such that $\rho \leq \sigma$, $\sigma \in \text{FSO}(\lambda)$. Then $\sigma \in \text{FSC}(\lambda)$ and so $\text{Scl}(\rho) \leq \text{Scl}(\sigma) = \sigma$. So, $\rho \in \text{FGSC}(\lambda)$.

Conversely, let $\rho \in \text{FSO}(\lambda)$, since each F-subset of $\mathcal{A}_\lambda$ is Fg-semiclosed, then $\rho \in \text{FGSC}(\lambda) \Rightarrow \text{Scl}(\rho) \leq \rho$. Then $\rho = \text{Scl}(\rho)$. So, $\rho \in \text{FSC}(\lambda)$. Next, let $\rho \in \text{FSC}(\lambda)$, $\rho' \in \text{FSO}(\lambda)$. Then $\text{Scl}(\rho') \leq \rho \leq \text{Scl}(\rho')_{\lambda} = \text{Sint}(\rho)$. Then $\text{Sint}(\rho) = \rho$. So, $\rho \in \text{FSO}(\lambda)$. \hfill \Box

Theorem 5.8. For a F-ts ($\lambda, \delta$), the following statements are equivalent:

(i) For each $\rho \in \text{FSC}(\lambda)$ and each $\sigma \in \text{FGSO}(\lambda)$ such that $\rho \leq \sigma$, there exists $\zeta \in \text{FSO}(\lambda)$ such that $\rho \leq \zeta \leq \text{Scl}(\zeta) \leq \sigma,$

(ii) For each $\rho \in \text{FSC}(\lambda)$ and each $\sigma \in \text{FGSC}(\lambda)$ such that $\sigma \leq \rho$, there exists $\zeta \in \text{FSC}(\lambda)$ such that $\sigma \leq \zeta \leq \text{Scl}(\zeta) \leq \rho.$

Proof. (i) $\Rightarrow$ (ii). Let $\sigma \in \text{FSC}(\lambda)$, $\rho \in \text{FSC}(\lambda)$ such that $\sigma \leq \rho$. Then $\rho' \leq \sigma'$. Since $\sigma' \in \text{FGSO}(\lambda)$, $\rho' \in \text{FSO}(\lambda)$. By (i), $\exists \zeta \in \text{FSO}(\lambda)$ such that $\rho' \leq \zeta \leq \text{Scl}(\zeta) \leq \sigma' \Rightarrow \sigma \leq \text{Scl}(\zeta)'_{\lambda} \leq \rho$. Put $(\text{Scl}(\zeta)')_{\lambda} = \eta$, then $\eta \in \text{FSO}(\lambda)$ and $\sigma \leq \eta \leq \text{Scl}(\eta) \leq \rho$.

(ii) $\Rightarrow$ (i): is similar to (i) $\Rightarrow$ (ii). \hfill \Box

Definition 5.9. A F-ts($\lambda, \delta$) is said to be

(i) FGS-T$_0$ if for each $x, y \in \text{Pt}(\lambda)$ with distinguished support points $x \neq y$, there exist $\zeta \in \text{FGSO}(\lambda)$ such that $x \in [\zeta, \zeta \text{^g} y, [\lambda]$ or $y \in [\zeta, \zeta \text{^g} x, [\lambda]$.

(ii) FGS-T$_1$ if for each $x, y \in \text{Pt}(\lambda)$ with distinguished support points $x \neq y$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $x \in [\zeta, \zeta \text{^g} y, [\lambda]$ and $y \in [\eta, \eta \text{^g} x, [\lambda]$.

(iii) FGS-T$_2$ if for each $x, y \in \text{Pt}(\lambda)$ with distinguished support points $x \neq y$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $x \in [\zeta, \zeta \text{^g} y, [\lambda]$ and $\zeta \text{^g} \eta \in [\lambda]$. 
(iv) FGS-T$_2$ if for each $x_r, y_s \in \mathcal{P}(\lambda)$ with distinguished support points $x \neq y$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta, y_s \in \eta$ and $\text{Scl}(\zeta) \cap \text{Scl}(\eta)[\lambda]$.

(v) FGS-R$_0$ if for each $x_r, y_s \in \mathcal{P}(\lambda)$ with distinguished support points $x \neq y$ and each $\zeta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta, \zeta \cap \text{Scl}_r[y_s][\lambda]$, there exists $\eta \in \text{FGSO}(\lambda)$ such that $y_s \in \eta, \eta \cap \text{Scl}_r[y_s][\lambda]$.

(vi) FGS-R$_1$ if for each $x_r, y_s \in \mathcal{P}(\lambda)$ with distinguished support points $x \neq y$ and each $\zeta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta, \zeta \cap \text{Scl}_r[y_s][\lambda]$, there exist $\rho, \sigma \in \text{FGSO}(\lambda)$ such that $x_r \in \rho, y_s \in \sigma$ and $\rho \cap \text{Scl}_r[y_s][\lambda]$.

**Remark 5.10.** From the above definition one may notice that:

(i) FGS-T$_{32} \Rightarrow$ FGS-T$_2 \Rightarrow$ FGS-T$_1 \Rightarrow$ FGS-T$_0$.

(ii) FS-T$_i \Rightarrow$ FGS-T$_i$, $i = 0, 1, 2, 2\frac{1}{2}$.

(iii) FGS-R$_1 \Rightarrow$ FGS-R$_0$.

(iv) FS-R$_i \Rightarrow$ FGS-R$_i$, $i = 0, 1$.

**Example 5.11.** Let $X = \{x, y\}$, $\lambda = x_{0.7} \lor y_{0.6}$, $\delta = \{0, x_{0.7} \lor y_{0.4}, y_{0.4}, \lambda\}$. It is clear that, the F-ts $(\lambda, \delta)$ is FGS-T$_0$, but not FGS-T$_1$.

**Theorem 5.12.** If a F-ts $(\lambda, \delta)$ is FGS-T$_0$ and FGS-R$_j$, then $(\lambda, \delta)$ is a FGS-T$_2$ space.

**Definition 5.13.** A F-ts $(\lambda, \delta)$ is said to be FG-semiregular if for each $x_r \in \mathcal{P}(\lambda)$, $\sigma \in \delta'$ such that $\sigma \cap \text{Scl}_r[y_s][\lambda]$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta, \sigma \leq \eta$ and $\zeta \cap \text{Scl}_r[y_s][\lambda]$.

**Remark 5.14.** FG-semiregular $\Rightarrow$ FG-semiregular.

**Theorem 5.15.** In a F-ts $(\lambda, \delta)$, the following statements are equivalent:

(i) $(\lambda, \delta)$ is FG-semiregular.

(ii) For each $x_r \in \mathcal{P}(\lambda)$ and each $\sigma \in \delta'$ such that $\sigma \cap \text{Scl}_r[y_s][\lambda]$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta, \sigma \leq \eta$ and $\text{Scl}(\zeta) \cap \text{Scl}(\eta)[\lambda]$.

(iii) For each $x_r \in \mathcal{P}(\lambda)$ and each $\rho \in \delta$ such that $x_r \in \rho$, there exists $\zeta \in \text{FGSO}(\lambda)$ such that $x_r \in \zeta \leq \text{Scl}(\zeta) \leq \rho$.

**Proof.** One can prove it by Theorem 5.5. $\square$

**Definition 5.16.** A F-ts $(\lambda, \delta)$ is said to be FG-seminormal if for each $\sigma_1, \sigma_2 \in \delta'$ such that $\sigma_1 \cap \text{Scl}_2[y_s][\lambda]$, there exist $\zeta, \eta \in \text{FGSO}(\lambda)$ such that $\sigma_1 \leq \zeta, \sigma_2 \leq \eta$ and $\zeta \cap \text{Scl}_2[y_s][\lambda]$. 
Theorem 5.17. A F-ts \((\lambda, \delta)\) is FG-seminormal iff for each \(\sigma \in D', \rho \in \delta\) such that \(\sigma \leq \rho\), there exists \(\zeta \in FGSO(\lambda)\) such that \(\sigma \leq \zeta \leq \text{Scl}(\zeta) \leq \rho\).

Remark 5.18. F-seminormal \(\Rightarrow\) FGS-seminormal.

6. Semiconnectedness

Definition 6.1. Let \((\lambda, \delta)\) be a F-ts and \(\zeta, \eta \in \mathcal{A}_\lambda\). Then \(\zeta\) and \(\eta\) are said to be

(i) F-semiseparated if \(\text{Scl}(\eta) \land \zeta = 0\) and \(\eta \land \text{Scl}(\zeta) = 0\).

(ii) F-\(\omega\)-semiseparated \(\text{Scl}(\eta) \hat{q}\zeta[\lambda] \land \eta \hat{q}\text{Scl}(\zeta)[\lambda]\).

\((\lambda, \delta)\) is said to be F-semiconnected if no proper non-zero maximal F-semiseparated \(\rho, \sigma \in \mathcal{A}_\lambda\) such that \(\zeta = \rho \lor \sigma\).

Remark 6.2. F-semiseparated \(\Rightarrow\) F-\(\omega\)-semiseparated

Theorem 6.3. Let \((\lambda, \delta)\) be a F-ts, \(\zeta, \eta, \rho \in \mathcal{A}_\lambda, \zeta\) and \(\eta\) are F-semiseparated. Then \(\zeta \land \rho, \eta \land \rho\) are F-semiseparated

Theorem 6.4. Let \((\lambda, \delta)\) be a F-ts and \(\zeta, \eta \in \mathcal{A}_\lambda\). Then:

(i) If \(\zeta, \eta\) are F-\(\omega\)-semiseparated and \(\rho, \sigma \in \mathcal{A}_\lambda\) such that \(\rho \leq \zeta, \sigma \leq \eta\), then \(\rho\) and \(\sigma\) are also F-\(\omega\)-semiseparated.

(ii) If \(\zeta \hat{q}\eta[\lambda]\) and either \(\zeta, \eta \in FSO(\lambda)\) or \(\zeta, \eta \in FSC(\lambda)\), then \(\zeta\) and \(\eta\) are F-\(\omega\)-semiseparated.

(iii) If \(\zeta \hat{q}\eta[\lambda]\) and either \(\zeta, \eta \in FSO(\lambda)\) or \(\zeta, \eta \in FSC(\lambda)\), \(\rho(\zeta) = \zeta \land \rho(\eta) = \eta \land \zeta\), then \(\rho\) and \(\eta\) are F-\(\omega\)-semiseparated.

Theorem 6.5. The F-subsets \(\zeta, \eta\) of \(\mathcal{A}_\lambda\) are F-\(\omega\)-semiseparated iff there exist \(\rho, \sigma \in FSO(\lambda)\) such that \(\zeta \leq \rho, \eta \leq \sigma, \zeta \hat{q}\sigma[\lambda]\) and \(\eta \hat{q}\rho[\lambda]\).

Proof. \(\forall F-\omega\)-semiseparated sets \(\zeta\) and \(\eta\) of \(\mathcal{A}_\lambda\), \(\eta \leq (\text{Scl}(\zeta))_\lambda = \rho\) and \(\zeta \leq (\text{Scl}(\eta))_\lambda = \zeta\), then \(\rho, \sigma \in \text{FSO}(\lambda), \zeta \hat{q}\rho[\lambda]\) and \(\eta \hat{q}\sigma[\lambda]\).

Conversely, let \(\rho, \sigma \in \text{FSO}(\lambda)\) such that \(\zeta \leq \rho, \eta \leq \sigma, \zeta \hat{q}\sigma[\lambda]\) and \(\eta \hat{q}\rho[\lambda]\). Then \(\zeta \leq \sigma\) and \(\eta \leq \rho\) \(\Rightarrow\) \(\text{Scl}(\zeta) \leq \sigma[\lambda]\) and \(\text{Scl}(\eta) \leq \rho[\lambda]\) \(\Rightarrow\) \(\text{Scl}(\zeta) \hat{q}\eta[\lambda]\) and \(\text{Scl}(\eta) \hat{q}\zeta[\lambda]\). So, \(\zeta\) and \(\eta\) are F-\(\omega\)-semiseparated.

Definition 6.6. A F-ts \((\lambda, \delta)\) is said to be F-semiconnected if it has no proper non-zero maximal F-semiclopen set.
Definition 6.7. $\zeta \in \mathcal{A}_k$ is said to be F-semiconnected if $(\zeta, \delta_\zeta)$ is F-semiconnected.

Remark 6.8. F-semiconnected $\Rightarrow$ F-connected.

Example 6.9. Let $X = \{x, y, z\}$, $\lambda = x_{0.8} \cup y_{0.6} \cup z_{0.6}$, $\delta = \{0, x_{0.6} \cup z_{0.6}, x_{0.6} \cup y_{0.6}, y_{0.6} \cup z_{0.6}, y_{0.6}, z_{0.6}, 0, \lambda\}$. It is clear that the F-ts $(\lambda, \delta)$ is F-connected but not F-semiconnected.

Remark 6.10. The F-semiclosure of a maximal F-semiconnected set of a F-ts $(\lambda, \delta)$ may not be F-semiconnected as shown by the following example.

Example 6.11. Let $X = \{x, y\}$, $\lambda = x_{0.7} \cup y_{0.6}$, $\delta = \{0, x_{0.35}, y_{0.35}, 0.35, x_{0.35} \cup y_{0.6}, \lambda\}$. It is clear that, $y_{0.6}$ is a maximal F-semiconnected, but $\text{Scl}(y_{0.6})$ is not F-semiconnected.

Theorem 6.12. Let $(\lambda, \delta)$ be a F-ts. Then the following statements are equivalent:

(i) $\zeta$ is F-semiconnected, $\zeta \in \mathcal{A}_k$.
(ii) $\forall \rho, \sigma \in \mathcal{A}_k$ are F-semiseparated, $\zeta \leq \rho \cup \sigma \Rightarrow \zeta \leq \rho$ or $\zeta \leq \sigma$.
(iii) $\forall \rho, \sigma \in \mathcal{A}_k$ are F-semiseparated, $\zeta \leq \rho \cup \sigma \Rightarrow \zeta \wedge \rho = 0$ or $\zeta \wedge \sigma = 0$.

Theorem 6.13. Let $\eta \in \mathcal{A}_k$ such that for each F-points $x_r, y_s \in \eta$, there exists a maximal F-semiconnected subset $\zeta$ such that $x_r, y_s \in \zeta \subseteq \eta$. Then $\eta$ is F-semiconnected.

Theorem 6.14. If $\{\zeta_i : i \in J\}$ is a family of F-semiconnected of $\mathcal{A}_k$ such that $\zeta_i \wedge \zeta_j \neq 0$, $\forall i, j \in J$, then $\zeta = \cup\{\zeta_i : i \in J\}$ is F-semiconnected.

Theorem 6.15. If $f : (\lambda, \delta) \rightarrow (\mu, \delta')$ is F-irresolute and if $\zeta$ is F-semiconnected of $\lambda$, then so is $f^{-1}(\zeta)$ of $\mu$.

Proof. Suppose $f^{-1}(\zeta)$ is not F-semiconnected of $\mu$. Then $f^{-1}(\zeta)$ has a non-zero proper maximal F-semiclopen set $\rho$ of $\mu$. Hence $f^{-1}(\rho)$ is a non-zero proper maximal F-semiclopen set of $(\zeta, \delta_\zeta)$ which contradicts that $\zeta$ is F-semiconnected. So, $f^{-1}(\zeta)$ is F-semiconnected.

7. Semicompactness

Definition 7.1. $(\lambda, \delta)$ is said to be F-semicompact if each F-semiopen cover of $\lambda$ has a finite subcover.
Remark 7.2. F-semicompact \( \Rightarrow \) F-compact.

**Definition 7.3.** \( \zeta \) is F-semicompact if \( (\zeta, \delta_{\zeta}) \) is F-semicompact, \( \zeta \in \mathcal{A}_{\lambda} \).

Remark 7.4. Let \( (\lambda, \delta) \) be a F-ts. Then:

(i) A F-semiclosed subset of a F-semicompact space may not be F-semicompact.

(ii) A F-semicompact subset of FGS-T_2 may not be F-semiclosed.

**Example 7.5.** Let \( X = \{x^1, x^2, x^3, \ldots, x^n, \ldots\} \), \( \lambda = 0.6 \), \( \delta \) is generated by \( \{0, \lambda\} \cup \{x^j_{0.4}: j = 1, 2, 3, \ldots\} \). Then the F-ts \( (\lambda, \delta) \) is F-compact but not F-semicompact. And \( \sigma = 0.2 \) is a F-semiclosed set of \( (\lambda, \delta) \) but not F-semicompact.

**Theorem 7.6.** If \( f : (\lambda, \delta) \rightarrow (\mu, \delta^*) \) is F-irresolute and \( \zeta \) is F-semicompact of \( \lambda \), then so is \( f^{-1}(\zeta) \) of \( \mu \).

**References**


